Sirindhorn International Institute of Technology

Thammasat University at Rangsit

School of Information, Computer and Communication Technology

ECS 455: Problem Set 4

Semester/Year: **2/2012**
Course Title: Mobile Communications
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Due date: Feb 4, 2013 (Monday), 8:50AM

1. Consider Global System for Mobile (GSM), which is a TDMA/FDD system that uses 25 MHz *only one direction* *simplex channel*
   for the forward link, which is broken into radio channels of 200 kHz. If 8 speech channels are
   supported on a single radio channel, and if no guard band is assumed, find the number of
   simultaneous users that can be accommodated in GSM. ← *Here, we ignore the gain from frequency reuse.*

   Solution *number of radio channels* *number of speech channels per radio channel (by TDMA)*

   $$\frac{25 \times 10^6}{200 \times 10^3} \times 8 = 1000 \text{ simultaneous users.}$$

2. Draw the **complete** state diagrams for linear feedback shift registers (LFSRs) using the
   following polynomials. Does either LFSR generate an m-sequence?

Solution

(a) $1+x^2+x^5$	(b) $1+x+x^2+x^5$	(c) $1+x+x^2+x^4+x^5$
(a) The LFSR will cycle through the following states:	(b) The LFSR will cycle through one of the cycles of states below. The initial state determine which cycle it will go through.	(c) The LFSR will cycle through the following states:

(a) $1+x^2+x^5$ — The LFSR will cycle through the following states:

```
1 0 0 0 0
0 1 0 0 0
1 0 1 0 0
0 1 0 1 0
1 0 1 0 1
1 1 0 1 0
1 1 1 0 1
0 1 1 1 0
1 0 1 1 1
1 1 0 1 1
0 1 1 0 1
0 0 1 1 0
0 0 0 1 1
1 0 0 0 1
1 1 0 0 0
1 1 1 0 0
1 1 1 1 0
1 1 1 1 1
0 1 1 1 1
0 0 1 1 1
1 0 0 1 1
1 1 0 0 1
0 1 1 0 0
1 0 1 1 0
0 1 0 1 1
0 0 1 0 1
1 0 0 1 0
0 1 0 0 1
0 0 1 0 0
0 0 0 1 0
0 0 0 0 1
```

(b) $1+x+x^2+x^5$ — The LFSR will cycle through one of the cycles of states below. The initial state determine which cycle it will go through.

Cycle #1:
```
1 0 0 0 0
1 1 0 0 0
0 1 1 0 0
1 0 1 1 0
1 1 0 1 1
1 1 1 0 1
1 1 1 1 0
0 1 1 1 1
0 0 1 1 1
1 0 0 1 1
0 1 0 0 1
0 0 1 0 0
0 0 0 1 0
0 0 0 0 1
```

Cycle #2:
```
0 0 0 1 1
1 0 0 0 1
0 1 0 0 0
1 0 1 0 0
1 1 0 1 0
0 1 1 0 1
0 0 1 1 0
```

Cycle #3:
```
0 0 1 0 1
1 0 0 1 0
1 1 0 0 1
1 1 1 0 0
0 1 1 1 0
1 0 1 1 1
0 1 0 1 1
```

Cycle #4:
```
0 1 0 1 0
1 0 1 0 1
```

Cycle #5:
```
1 1 1 1 1
```

(c) $1+x+x^2+x^4+x^5$ — The LFSR will cycle through the following states:

```
1 0 0 0 0
1 1 0 0 0
0 1 1 0 0
1 0 1 1 0
0 1 0 1 1
1 0 1 0 1
0 1 0 1 0
0 0 1 0 1
1 0 0 1 0
0 1 0 0 1
0 0 1 0 0
0 0 0 1 0
1 0 0 0 1
0 1 0 0 0
1 0 1 0 0
1 1 0 1 0
1 1 1 0 1
1 1 1 1 0
1 1 1 1 1
0 1 1 1 1
1 0 1 1 1
1 1 0 1 1
0 1 1 0 1
0 0 1 1 0
1 0 0 1 1
1 1 0 0 1
1 1 1 0 0
0 1 1 1 0
0 0 1 1 1
0 0 0 1 1
0 0 0 0 1
```

The polynomial $1+x^2+x^5$ and $1+x+x^2+x^4+x^5$ from part (a) and (c) generate m-sequences. (Their states go thorough cycle of size 2^5-1)

3. Use any resource, find <u>all</u> primitive polynomials of degree 6 over GF(2). Indicate your reference.

    Solution

Primitive Polynomials
$x^6 + x^1 + 1$
$x^6 + x^5 + x^2 + x^1 + 1$
$x^6 + x^5 + x^3 + x^2 + 1$
$x^6 + x^4 + x^3 + x^1 + 1$
$x^6 + x^5 + x^4 + x^1 + 1$
$x^6 + x^5 + 1$

    Source: http://www.theory.cs.uvic.ca/~cos/gen/poly.html

4. See the hand-written solution at the end.

5. In CDMA, each bit time is subdivided into *m* short intervals called **chips**. We will use 8 chips/bit for simplicity. Each station is assigned a unique 8-bit code called a **chip-sequence**. To transmit a 1 bit, a station sends its chip sequence. To transmit a 0 bit, it sends the one's complement[1] of its chip sequence.

    Here are the binary chip sequences for four stations:

    A: 0 0 0 1 1 0 1 1
    B: 0 0 1 0 1 1 1 0
    C: 0 1 0 1 1 1 0 0
    D: 0 1 0 0 0 0 1 0

    For pedagogical purposes, we will use a bipolar notation **with binary 0 being -1 and binary 1 being +1**. In which case, during each bit time, a station can transmit a 1 by sending its chip sequence, it can transmit a 0 by sending the negative of its chip sequence, or it can be silent and transmit nothing. We assume that all stations are synchronized in time, so all chip sequences begin at the same instant.

    When two or more stations transmit simultaneously, their bipolar signals add linearly.

    a. Suppose that A, B, and C are simultaneously transmitting 0 bits. What is the resulting (combined) bipolar chip sequence?

    b. Suppose the receiver gets the following chips: (-1 +1 -3 +1 -1 -3 +1 +1).
        Which stations transmitted, and which bits did each one send?

    Solution

```
%Chip sequences
C = [0 0 0 1 1 0 1 1; 0 0 1 0 1 1 1 0; 0 1 0 1 1 1 0 0; 0 1 0 0 0 0 1 0];
C = 2*C-1; %Change to bipolar form
```

---

[1] You should have seen the "one's complement" operation in your "digital circuits" class.

```
% Part a
m = [-1 -1 -1 0]  %message to transmit
x = %%%%%%%%HELP ME%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

% Part b
r = [-1 1 -3 1 -1 -3 1 1] ;
m_decoded = 1/8* %%%%%%%%HELP ME%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%This gives [mA mB mC mD]' in bipolar form;
%The value is 1 if 1 was transmitted. The value is 0 if nothing was
%transmitted. The value is -1 if 0 was transmitted.
```
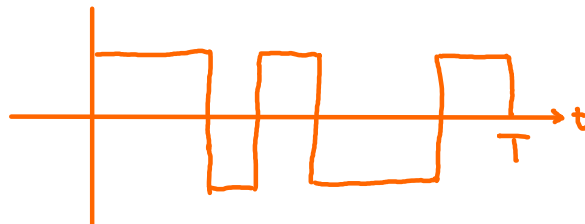
Use the above MATLAB code with x = m*C; and m_decoded = (C*r')/8;

(a) [3 1 1 -1 -3 -1 -1 1]

(b) [1 -1 0 1]'; Hence, A and D sent 1 bits, B sent a 0 bit, and C was silent.

We will try to find waveforms that simply oscillate between positive and negative values:

Now, to make it orthogonal to $C_1(t)$, the positive portion of the graph must be equal to the negative portion of the graph.

(So, the above waveform does not work.)

It should also be orthogonal to $C_2(t)$. One way to make this happen is to divide the interval $[0, T]$ into many parts, say 4 parts.
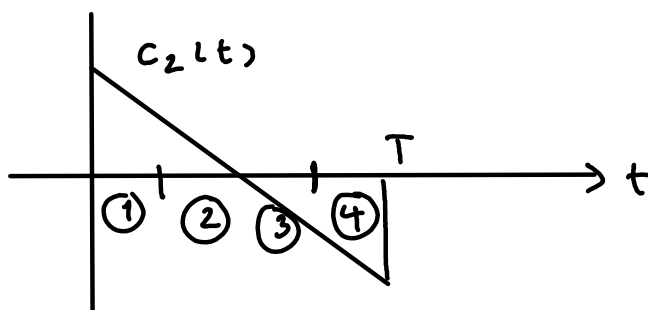
To ensure orthogonality, we may require

and sections ① and ④ to be multiplied by the same sign

sections ② and ③ to be multiplied by the same sign.

From this, there are 4 options

①,④ × ">0"   ②,③ × ">0"

①,④ × " < 0 " ②,③ × " < 0 "
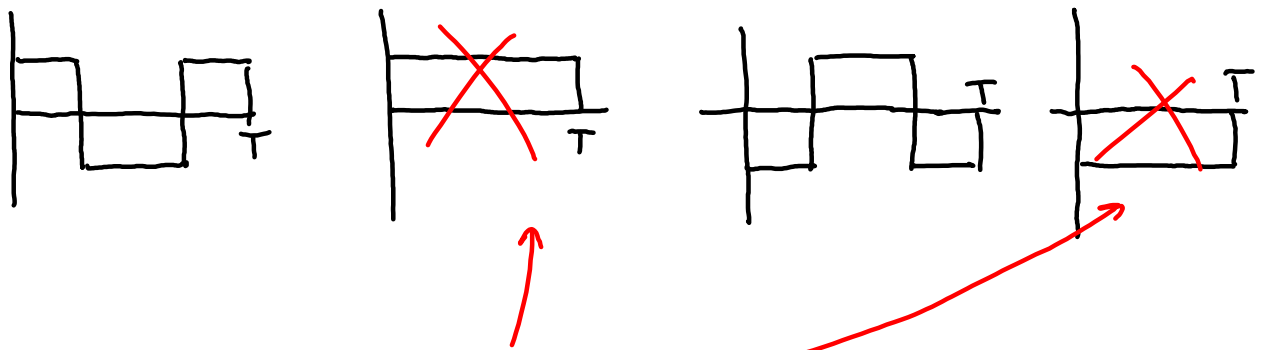
not orthogonal to $c_1(t)$

The two new waveforms are orthogonal to both $c_1(t)$ and $c_2(t)$ but they are not orthogonal to one another.

We will keep only one:

$c_3(t) =$
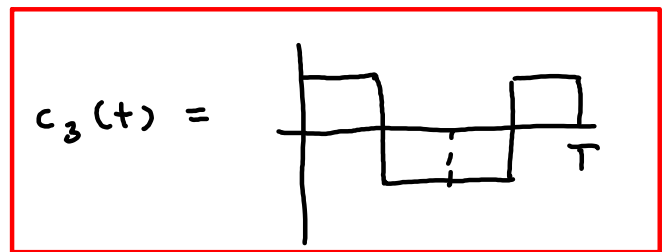
At this point, we have

$$\int_0^T c_1(t) c_3(t)\, dt = \int_0^T c_2(t) c_3(t)\, dt = 0$$

To find $c_4(t)$,

we may have to further divide the interval

To do this,
an easy way is to split each section into positive and negative sub-section:

This way, the new waveform will be orthogonal to both $C_1(t)$ and $C_3(t)$.

As drawn it is not orthogonal to $C_2(t)$ so we will need to switch the sign.

Note that $C_2(t)$ is symmetric around $\frac{T}{2}$.

Hence, our $C_4(t)$ will be " " " as well; so that their product will still have equal positive and negative areas.

One such option is:

$$C_4(t) = \quad \text{[square wave diagram]} \quad T$$
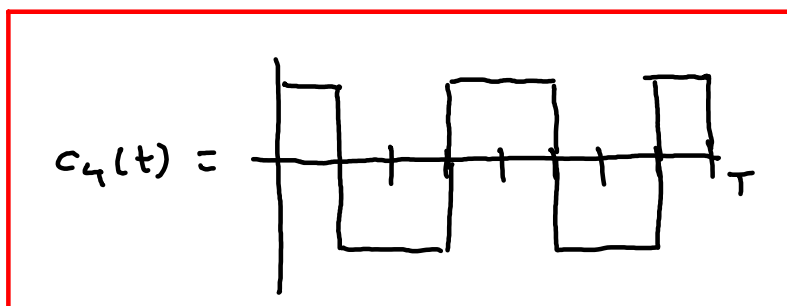
The final step is to check that

$$\boxed{\int_0^T c_1(t)c_2(t)\,dt} = \int_0^T c_1(t)\,c_3(t)\,dt = \int_0^T c_1(t)c_4(t)\,dt = 0$$

$$\int_0^T c_2(t)\,c_3(t)\,dt = \int_0^T c_2(t)\,c_4(t)\,dt = \int_0^T c_3(t)\,c_4(t)\,dt = 0.$$

This is given to be $=0$.

The other integrations are $=0$ by construction.